



## Teaching Math on an Oscilloscope, Part III: FFT “Leakage”

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### Objective:

To show mathematical concepts in real time: integration, differentiation and FFT conversions. Note that this experiment is a way for the math, physics and engineering departments to work together to show students some “real” applications of math.

### Equipment:

- Agilent 33120A Synthesized Function/Arbitrary Waveform Generator
- Agilent 54600B 100 MHz Digital Oscilloscope with
- Agilent 54657A FFT and GPIB plug-on module
- Agilent BenchLink/Suite software
- PC with GPIB capability

### Prerequisites:

A very basic understanding of calculus and Fourier Transforms. Math Functions on a Scope, Parts I and II.

In part II of this experiment, we covered basic FFT functions of various waveforms. This experiment will give us insight as to just how the FFT function is performed inside the oscilloscope.

### Leakage

When the scope computes the FFT of a waveform, using a rectangular “window”, it assumes that the exact same waveform that is on the screen is one cycle of the complete waveform, and that waveform continues to infinity in both directions. For example, using a rectangular window and a “perfect” sine wave, we get a very good FFT [See FIG 15]:

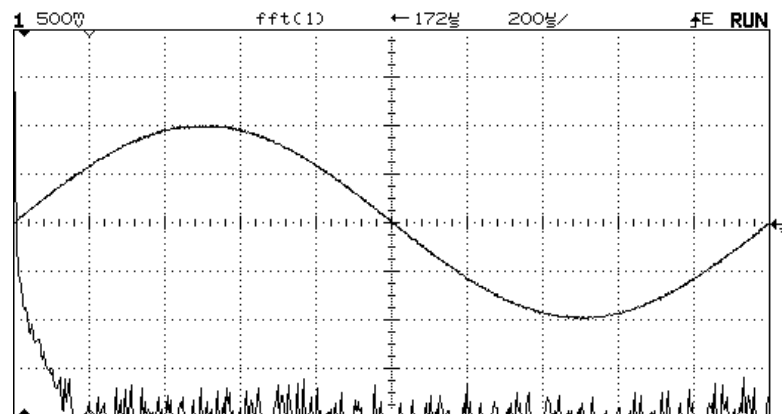


FIG 15: “Good” sine wave with narrow FFT spike.

Note how narrow the FFT is. There is very little doubt that it is composed mainly of a single frequency.



Now look what happens when we put slightly MORE than a full cycle on the screen:

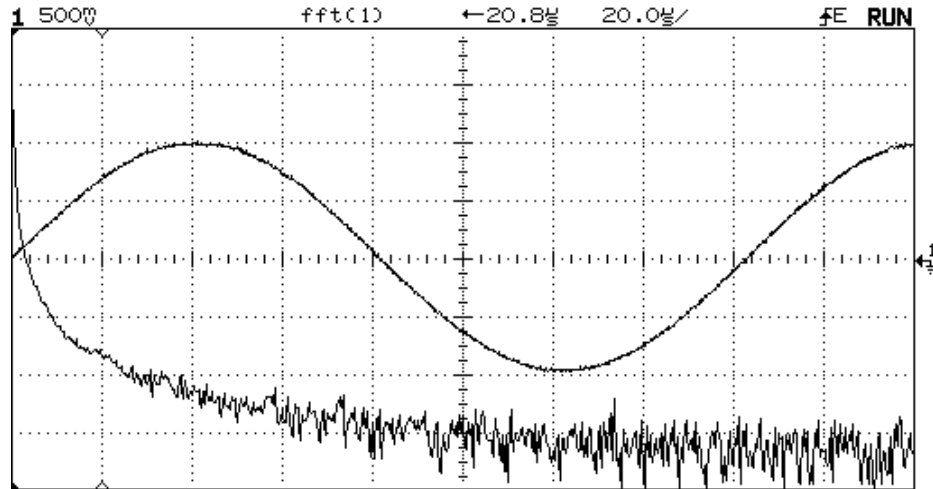


FIG 16: 450 degrees of a sine wave

**Leakage (cont'd):**

In FIG 16, check the width of the FFT spectrum compared to that of FIG 15. Look one major division below the center line, and compare the width of the two FFT's. The FFT in FIG 16 is much wider. That's called "leakage", and is caused by the scope "thinking" that the sine wave is no longer a sine wave, but a waveform that goes for 1 1/4 cycles and then repeats itself. What would the waveform in Fig 16 look like if we were to replicate it for more than one cycle? We can show this by the following process:

- Upload the FIG 16 waveform from the scope using Agilent BenchLink/Scope software
- Edit/Copy the waveform into Agilent BenchLink/Arb software
- Download the waveform to the arbitrary waveform memory of the Agilent 33120A Function/Arb Generator using BenchLink/Arb software
- Set the scope to show two or more cycles of the new waveform.
- Compute the FFT with the scope
- The scope now sees exactly what it "thought" it saw in FIG 16. Indeed, the FFT is nearly identical to the FFT in FIG 16. [See FIG 17]

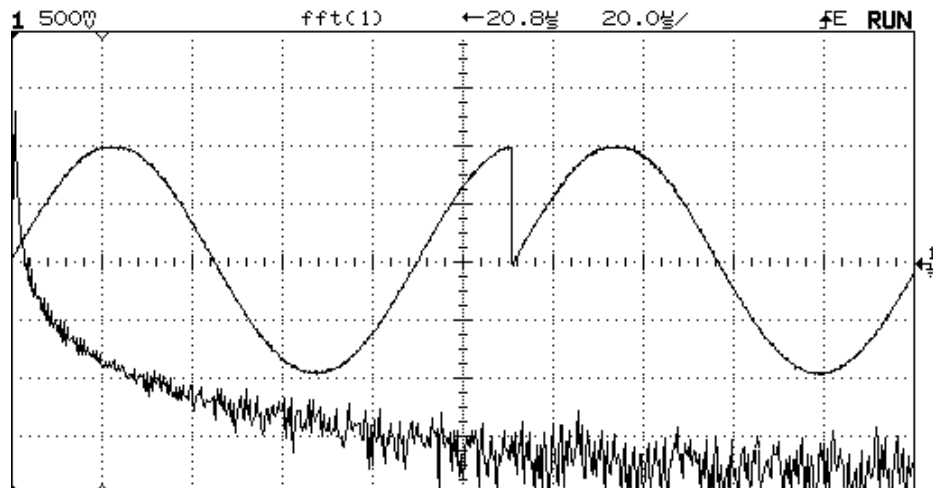


FIG 17: 450 degree "sine" wave, replicated on Agilent 33120A



**Note how closely this FFT matches that of FIG 16. The “leakage” in the FFT spectra in FIG 16 is caused by the scope “assuming” that the sine wave indeed looks like the waveform in FIG 17.**

If you need to compare the two different FFT’s directly on the scope screen, simply save the first one to TRACE 1 using the TRACE keys on the Agilent 54600-series scope. You can then overlay the second FFT to compare them.

**The FFT in FIG 17 was done with a RECTANGULAR window function. What happens if you change the WINDOW from RECTANGULAR to HANNING? Explain the difference.**

For more on FFT **WINDOWING**, consult:

- Agilent product note 54600-3: FFT Lab Experiments Notebook [free with purchase of Agilent 54657A and Agilent 54659B FFT modules].
- Agilent product note 54600-4: Using the FFT in Agilent 54600-Series Oscilloscopes

Parts I,II and III of this experiment are but a few examples of how an educator can use the built-in functions of Agilent Basic Instruments to demonstrate basic mathematical principles. Obviously, there are many more features in these instruments that we could use to accomplish the same purpose, using signals such as FM, AM, Burst, Arbitrary waveforms, Cardiac waveforms and condition simulation, Windowing, Noise, etc.

We welcome your comments on these exercises. You can reach Agilent at the Educator’s Corner Website, <http://www.educatorscorner.com>

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